Equivalent Circuit Model of Supercapacitor for Self-Discharge Analysis - A Comparative Study

Pankaj Saha  
Department of EIE  
NIT Silchar  
Assam, India  
send2pankajsaha@gmail.com

Munmun Khanra, member IEEE  
Department of EIE  
NIT Silchar  
Assam, India  
munnunkhanra@gmail.com

Abstract—Self-discharge of supercapacitor is an important phenomenon that needs to be considered carefully, especially for the applications like wireless sensor networks, memory backup systems, etc. Different types of models have been proposed by the various researchers to capture the supercapacitor dynamics. In this paper, two electrical equivalent circuit models, variable leakage resistance (VLR) model and charge redistribution based model, have been considered for comparative study. Both the models have been analyzed based on the charge and long term self-discharge responses obtained through experimentation and simulation. The device under test is the Maxwell BCAP0100 P270 T07 supercapacitor. First, it has been charged at 2A constant current to its rated voltage; then, has been left for self-discharge. The terminal voltage has been collected for total charging time as well as self-discharge for 8000 seconds. The data have been acquired using National Instruments (NI) hardware and LabVIEW software. Based on the case study used in this paper, it is observed that the charge redistribution based model is better able to capture the self-discharge phenomenon. However, the error in the VLR model may not be insignificant always.

Keywords — charge redistribution; electrical equivalent circuit; self discharge; supercapacitor; ultracapacitor.

I. INTRODUCTION

Modern world is moving towards using renewable energy; and energy storage devices play an important role in effective utilization of the same. Since long back, the commonly used energy storage device has been the battery. The supercapacitors, or ultracapacitors or electric double layer capacitors have been included in the list of top interest of the researchers, since a few decades. The important characteristics of supercapacitor are less hazardous [1], fast charging capabilities, high power density [2] (in order of 103 W/Kg), and long life time (50,000 - 1,000,000 cycles). These important features have made supercapacitors popular in the applications like hybrid electric vehicle, energy harvesting, wireless sensor network, memory backup system, power bank for improvement of battery performances [3-5]. However, supercapacitor has some disadvantages like having less energy density and higher self-discharge at no load condition in compared to battery. Self-discharge becomes a serious issue when long term response of supercapacitor is important like in case of power supply to wireless sensor network, memory backup system, etc. The reasons behind the self-discharge are redistribution of charge into the porous structured carbon electrode by diffusion and leakage current flow through the ohmic path between electrode-electrolyte interfaces.

The modelling of supercapacitor helps to get an idea about the terminal voltage, after long time duration at no load condition, before using the component in practical field for any application. A good amount of effort has been put on this topic [4-14]. In this paper, authors consider two existing models [7-8] which well account the response of a supercapacitor during self-discharge.

The remaining paper is structured as follows. Section-II describes the fundamental of supercapacitor. The self-discharge oriented electrical equivalent circuit models, considered in this study, are being discussed in Section-III. Section-IV contains the implementation of the considered models in both the simulation and experimentation. Section-V contains the results and discussion. Finally, Section-VI concludes the paper.

II. FUNDAMENTAL OF SUPERCAPACITOR

Whenever a charged surface comes in contact with any electrolyte solution, a potential difference created between the surface and the solution by means of attraction and repulsion of ions of opposite and like charges respectively. So an ion accumulation of opposite charges occurs at the vicinity of the surface–solution interface. This layer of opposite charges is called electric double layer (EDL).

In 1871, Helmholtz first explains the electric double layer which is known as Helmholtz model. According to Helmholtz model, the attracted ions of opposite charges are completely neutralized by the surface charges; the surface electrostatic force makes the ions immobilized and electric double form. However according to Gouy-Chapman theory on electric double layer, the ions are not immobilized on the surface as they are subjected to random thermal motion. From that point the concept of diffuse double layer comes. According to
which the ions, responsible for neutralizing the surface charges are not immobilized on the surface but they are spread out into solution. In 1924, Stern proposed a model of electric double layer which is a combination of both the Helmholtz model and Gouy-Chapman model i.e. some amount of the surface charges are neutralized by the immobilized ions but that is not sufficient to completely neutralize the whole charged surface. The rest amounts of charges are neutralized by diffused layer. Schematic of these three models are shown in Fig. 1.

The reason behind the diffusion of counter ions towards the electrode-electrolyte interface is the chemical potential or concentration gradient of ions. This concentration gradient is created due to the attraction of opposite charges from the nearby region of charged electrode in electrolyte. In addition to the concentration gradient, transport of ions towards the charged electrode surface may happen due to natural convection (transport of ions due to mechanical motion, vibration etc.) and effect of local heating too.

Electric double layer has two effects: 1) electrocapillarity, which is difficult to recognise in case of rigid electrode, 2) capacitive effect, based on which supercapacitor is developed. As mentioned earlier, the double charge layer is created at the electrode-electrolyte interface, which acts as storage of electric charge and behaves as a capacitor. The capacitance per unit area:

\[
C = \frac{dq}{d\phi}
\]  

(1)

‘q’ denotes the charge of the particle present in the electrolyte.

According to the Stern model, the total capacitance, \(C_T\) across a double layer is a combination of Helmholtz effect and Gouy-Chapman effect and can be defined as,

\[
C_T = C_H + \frac{\varepsilon_0\epsilon_0}{X_d}
\]  

(2)

where,

\[
X_d = \left[\frac{\varepsilon_0RT}{F^2}\right]^{1/2}
\]

\(\varepsilon\) : Dielectric constant of the medium.

\(\varepsilon_0\) : Permittivity of free space.

\(R\) : Gas Constant.

\(T\) : Absolute temperature.

\(F\) : Faraday constant.

\(I\) : Ionic strength.

The first term of the R.H.S of equation (2) denote the capacitance effect due to the Helmholtz layer and the second term denotes the Gouy-Chapman effect.

The distance between the parallel ionic layers is very small in the order of ionic radius (Angstrom). Therefore the capacitance of double layer capacitor is increased mainly by increasing the electrode surface area. Most commonly used electrode material for supercapacitor is activated carbon having specific surface area 1000-2000 m2/g, and specific capacitance is up to 500F/g, theoretically [16].

III. SELF-DISCHARGE ORIENTED ELECTRICAL EQUIVALENT CIRCUIT MODELS

Two electrical equivalent circuit models, the variable leakage resistance model and the charge redistribution based model, which properly account for the self-discharge phenomenon of supercapacitors, have been considered, in this paper, for a comparative analysis and are discussed below

A. Variable Leakage Resistance Model[7]

As mentioned previously, the self-discharge is an important phenomenon that occurs in supercapacitor. R. Faranda et al. at [6] proposed an equivalent circuit model namely ‘two branches model’ for describing the supercapacitor terminal behaviour at no load condition as shown in Fig. 2. Variable leakage resistance model is a modified form of that ‘two branches model’ where the researcher, H. Yang and Y. Zhang in [7] described a new method to characterize the leakage current flow of the supercapacitor, which is also responsible for self-discharge phenomenon, through a time dependent resistor as shown in Fig. 3. The first branch of the Variable leakage resistance model shown in Fig. 3 represents the immediate charge discharge effect of supercapacitor, whereas the second branch is for capturing the charge redistribution effect which can be observed up to three time constant after the charging of first branch is completed. The variable leakage resistance \(R\) (t) is used to show the decreasing probability of leakage current flow within the supercapacitor. As mentioned earlier, the leakage current is flowing due to ohmic path between electrode-electrolyte interfaces and charge diffusion, which decreases with time.
Assuming the charging current is flowing only through the first branch of the model, the model parameters can be identified based on the terminal voltage of supercapacitor with the help of the empirical formulae \[7\] given below:

\[
R_{\text{esr}} = \frac{\Delta V}{I_{\text{chg}}} \tag{4}
\]

\[
\Delta V = \frac{V_1 - V_2}{t_1 - t_2} \tag{5}
\]

\[
C_1 = \left[ \frac{t_1}{V_1} \frac{V_1 t_2 - t_1 V_2}{V_2 - V_1 t_2} \right] I_{\text{chg}} \tag{6}
\]

\[
K = 2 \left[ \frac{V_1 t_2 - t_1 V_2}{V_2 V_2 - V_1 V_2} \right] I_{\text{chg}} \tag{7}
\]

\[
Q_{\text{total}} = C_2 V_3 \tau + \left( C_1 + \frac{K}{2} V_3 \tau \right) V_3 \tau \tag{8}
\]

\[
\tau = R_2 C_2 \tag{9}
\]

where,

\(I_{\text{chg}}\) is charging current.

\(V_1\) is the voltage at time \(t_1\) during charging.

\(V_2\) is the voltage at time \(t_2\) during charging.

\(\tau\) is the time constant of the 2nd branch.

\(V_3\tau\) is the terminal voltage at time \(3\tau\).

The set of above equations can be used for identifying the parameters, except \(R(t)\), of the model in Fig. 3. Now, the time varying resistance \(R(t)\), also called leakage resistance can be identified by the Simulink model shown in Fig. 4.

---

### B. Equivalent Circuit Model Based on Charge Redistribution by Diffusion

In [8], V. Sedlakova et al. proposed a five parameter equivalent circuit model of supercapacitor for capturing the charge redistribution effect inside the component as shown in Fig. 5. The first branch represent the Helmholtz capacitance effect by the capacitor, \(Ch\). Unlike variable leakage resistance model, the leakage current which is flowing through the fixed load resistance \(R_{\text{load}}\) is constant, in this case. The charge redistribution effect is captured through the time dependent resistor \(R(t)\) in series with a diffusion capacitance \((Cd)\). The resistance \(R(t)\) shows the decreasing tendency of diffusion current flow with increase in time.

![Simulink model for identification of \(R(t)\) in Fig. 3](image4.png)

![Charge redistribution based model](image5.png)

Instead of any empirical equation like variable leakage resistance model, curve fitting method is used here for finding the value of time dependence of the resistor \(R(t)\) from the terminal voltage response of the supercapacitor. The self-discharge terminal voltage is fitted by an exponential function, where the exponent is proportional to the square root of time.
(t), which concluded that the time dependent resistor is a function of square root of time.

\[ R_t = a \sqrt{t_f - t_i} \]  \hspace{1cm} \text{(10)}

where,

‘a’ is a constant.
‘ti’ initial time when charging or discharging starts.
‘tf’ final time at which instant resistance will be calculated.

The other model parameters have been identified based on curve fitting of experimental data by analytical functions [8].

IV. IMPLEMENTATION OF THE TWO MODELS

In this section, the performance of the two selected models [7-8] have been demonstrated through the charge and self-discharge response of the Maxwell supercapacitor BCAP0100 P270 T07. The specification of the device under test is mentioned in Table I. In this line, first, the parameters of the two models have been identified through experimentation. Next, the identified models have been used to obtain the charge self-discharge response using MATLAB simulation; and the two simulated responses have been compared with the experimental response.

TABLE I: SPECIFICATION OF MAXWELL BCAP0100 P270 T07 SUPERCAPACITOR

<table>
<thead>
<tr>
<th>Rated voltage</th>
<th>Rated capacitance</th>
<th>Equivalent series resistance</th>
<th>Leakage current</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.7 V</td>
<td>100 F</td>
<td>15 mΩ</td>
<td>0.26 mA</td>
</tr>
</tbody>
</table>

A. Experimental Setup

The experimental setup, shown in Fig. 6, is consisting of Aplab L3202 DC regulated power supply, NI PXIe-6341X series multifunction DAQ, NI ELVIS II+ and NIPXIe-1071 system as a control computer. DC regulated power supply has been used at constant current mode for charging the supercapacitor. The regulated power supply can provide 0-2A constant current with a resolution of 10mA. The NI PXIe-6341X series multifunction DAQ and the NI ELVIS II+ has been used for collecting the supercapacitor terminal voltage and the charging current respectively. The whole measurement process was controlled by NIPXIe-1071 system based on LabVIEW software, 2013 version. The charging has been performed at 26.67C-rate i.e. 2A for 100F 2.7V supercapacitor. The charging current and terminal voltage response of the supercapacitor have been collected at 0.5Hz sampling rate for 8000 seconds.

B. Identification of the Model Parameters

The parameters of the two selected models have been identified based on the experimental data collected by using the hardware setup mentioned in previous subsection and the methods described in [7-8]. For variable leakage resistance model, the parameters have been identified using the equations (4)-(9) and the Simulink model in Fig. 4. The identified model parameters corresponding to the 100F 2.7V Maxwell supercapacitor are given in Table II.

TABLE II: VARIABLE LEAKAGE RESISTANCE MODEL PARAMETER CORRESPONDING TO MAXWELL BCAP0100 P270 T07 SUPERCAPACITOR

<table>
<thead>
<tr>
<th>Resr</th>
<th>C1</th>
<th>C2</th>
<th>K</th>
<th>R2</th>
</tr>
</thead>
<tbody>
<tr>
<td>18 mΩ</td>
<td>36.64 F</td>
<td>52.67 F</td>
<td>10.34</td>
<td>0.7 mΩ</td>
</tr>
</tbody>
</table>

The change in the leakage resistance with time is shown in Fig. 7.

The parameters of the charge redistribution based model have been identified using the ‘Trust-Region-Reflective’ algorithm of ‘Nonlinear least squares’ optimization method available with the MATLAB parameter estimation toolbox. The terminating conditions for the selected estimation process for finding the optimum model parameters are chosen to be the default values. The identified parameters values corresponding to the 100F 2.7V Maxwell capacitor are presented in Table III.
Fig. 7: Value of the leakage resistance at different time instant

Table III: Charge Redistribution Based Model Parameter Corresponding to Maxwell BCAP0100 P270 T07 Supercapacitor.

<table>
<thead>
<tr>
<th>Resr</th>
<th>Ch</th>
<th>Cd</th>
<th>R_load</th>
<th>a</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.135 Ω</td>
<td>101.4 F</td>
<td>12.65 F</td>
<td>5.1 KΩ</td>
<td>4.07</td>
</tr>
</tbody>
</table>

V. RESULTS AND DISCUSSION

A comparison has been done between the experimental output and the output of the identified models as discussed in previous section. The charge self-discharge responses are shown in Fig. 8. Fig. 9 demonstrates the error encountered by the two models identified for Maxwell BCAP0100 P270 T07 supercapacitor. The errors in the two identified models have been calculated in terms of the root means square error (RMSE), the sum of square error (SSE) and the maximum deviation between the experiment and model output. The calculated errors have been presented in Table IV.

Table IV: Comparison of Different Types of Error Present in the Two Model

<table>
<thead>
<tr>
<th>Model type</th>
<th>Variable leakage resistance model</th>
<th>Charge redistribution based model</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSE</td>
<td>3.2 %</td>
<td>0.67 %</td>
</tr>
<tr>
<td>SSE</td>
<td>4.1102</td>
<td>0.1775</td>
</tr>
<tr>
<td>Max. deviation</td>
<td>0.2317 V</td>
<td>0.1055 V</td>
</tr>
</tbody>
</table>

It can be observed from the Fig. 8, Fig. 9, and the data in Table IV that both the models are following the experimental output well and, thus, able to represent the long term response at no load condition. However, charge redistribution based model is better able to capture the transient as well as steady self-discharge (both the redistribution due to diffusion and the loss due to leakage resistance). This may be due to the fact that this model has considered the physics of the device in a more accurate manner in terms of Ch, the Helmholtz capacitance; and Cd, the diffusion capacitance.

Fig. 8: Experimental and models response of supercapacitor charging and self-discharging

Fig. 9: Error encountered by the two identified models

VI. CONCLUSION

A comparative analysis between two supercapacitor models, the variable leakage resistance model and the charge redistribution based model, intended for capturing the long term self-discharge response of supercapacitor, has been performed. The device under test is Maxwell BCAP0100
P270 T07 supercapacitor of 100F, 2.7 volt. The model parameters have been identified through experimentation. The experimental data and the simulated outputs have been compared. Based on the case study used in this paper, it is observed that both the models are following the long term self-discharge experimental data. However, the model in Fig. 5 shows more faithfulness compared to the model in Fig. 4. The RMSE due to charge redistribution model is 0.67%, whereas, the same due to the variable leakage resistance model is 3.2% for this particular case. However, more studies need to be performed for a strong conclusion.

References


